Interoperable Competencies Characterizing Learning Objects in Mathematics

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Abstract. Cognitive task analysis has been used in ITSs to predict students' performance, improve curricula and to determine appropriate feedback. Typically, the learning factors/knowledge components have been determined only for the use in *one* ITS or curriculum and therefore, general frameworks were not applied. Moreover, the result is sometimes rather unsystematic and not reusable across domains. However, for making learning environments interoperable and comparable and to be able to reuse learning objects, the competency hierarchies have to be usable for different learning environments and across domains. In this paper, we propose an approach to competencies represented as pairs of knowledge and cognitive process whose ontologies extend and revise existing taxonomies. A goal is to make these competencies a quasi-standard that enables interoperability and reuse. Moreover, we briefly describe, how the competency ontology can be employed for different purposes.

1 Introduction

The cognitive analysis of domains and tasks is a common requirement for designing an ITS and/or its content as well as for designing the structure of student models. The competencies resulting from this analysis (also called learning factors, knowledge elements, skills, or knowledge components) have been used to characterize tasks, exercises and exercise steps with the goal to choose appropriate exercises, and to improve a curriculum, e.g. in [1, 2, 3]. In pedagogical psychology, cognitive task analysis plays a role for diagnosing and evaluating students' learning progress [4].

As a general framework for such an analysis, Bloom devised his well-known taxonomy of educational objectives [5]. The cognitive objectives he addressed are general and domain independent and a relation to domain-specific settings is missing. More recent work in this field has extended Bloom's taxonomy [6].

As opposed to Bloom, the learning factor/knowledge component analysis for ITSs typically focuses on domain knowledge [2] and targets a set of knowledge elements for the usage in *one* ITS, sometimes even without a hierarchical structure.

[4] For instance, the knowledge components *integers* and *addition* in exercises of the ASSISTment system [5] denote elements of a domain ontology without explicitly addressing the cognitive or meta-cognitive processes needed in an exercise for integers or addition. However, for making learning environments interoperable and comparable and to be able to reuse learning objects and competency annotations, the competency hierarchies have to be usable across different learning environments and (partially) across domains.

In recent years, the appropriateness of the competencies resulting from the knowledge component analyses has been questioned wrt. the actual elements and their granularity [2, 7]. The main goal of the questioning was to improve the design of curricula and the prediction of students' performance. Now, we question the knowledge component/competency representation. The need for this arises from the aim of sharing content, reuse components, and services for learning environments and from the use of (standardized) metadata in web-based learning systems.

In this paper, we propose an approach that extends existing work, fits the needs of cognitive task analysis in various domains (e.g. fractions and calculus) and that is implemented in the ActiveMath platform. The main contributions are an ontology which refines a recent competency hierarchy that pair knowledge with cognitive processes, (partial) translations of previous taxonomies into the new ontology, and a demonstration of usages of the resulting competencies.

2 Previous Competency Systems

Learning objects such as exercises as well as their steps can be characterized (in technical terms: annotated) with the competencies required to solve a problem/succeed with a step.

For the ASSISTment Project Heffernan et al. have empirically determined so-called knowledge components (called skills) to characterize exercises and their steps [7]. These knowledge components include: *integers, addition, rounding, ordering numbers, reduce fractions, equivalence of fractions and decimal percents, equiliteral triangle, evaluating functions, finding percents, statistics, making sense of expressions and equations, order of operations, graph shape, reading graph, divide decimals, . . .* and have been defined by subject matter experts and by analyzing the Massachusetts curriculum. All of these knowledge components point to knowledge (a concept, rule, or procedure of the domain). Some of these knowledge components could also be interpreted as pointing to knowledge *and* a cognitive process.

Bloom [5] describes a hierarchy of educational goals that include:

- `Knowledge`: remembering; memorizing; recognizing; recall identification; recall of information

[4] Typically, the learning factors/knowledge components have been determined empirically by expert judgement for exercises or by analyzing curricula.

[5] Personal communication with Neil Heffernan

– `Comprehension`: interpreting; translating from one medium to another; describing in one's own words; organization and selection of facts and ideas
– `Application`: problem solving; applying information to produce some result; use of facts, rules and principles
– `Analysis`: subdividing something to show how it is put together; finding the underlying structure of a communication; identifying motives; separation of a whole into component parts
– `Synthesis`: creating a unique, original product that may be in verbal form or may be a physical object; combination of ideas to form a new whole
– `Evaluation`: making value decisions about issues; resolving controversies or differences of opinion; development of opinions, judgments or decisions

Anderson et al. [6] extends Bloom's taxonomy and pairs cognitive processes with knowledge to represent *competencies*. They introduce two different dimensions, the dimension of cognitive processes and the dimension of knowledge. Pairs of cognitive processes and knowledge elements form objectives (that constitute the basic building blocks of curricula). This was motivated by analyzing objectives listed in curricula, usually consisting of phrases such as "The student will learn to differentiate between rational numbers and irrational numbers" ([6], p. 5). They point out that such phrases typically are composed of a verb describing the intended cognitive process and one or more nouns referring to knowledge the students are supposed to acquire.

The competencies used for the PISA studies [8] include `think, argue, model, solve, represent, language, tools`. These top competencies have subcompetencies, e.g. `model` has the subcompetenices *decode* and *encode*.

The PISA competencies arise from the international discussion in mathematics education (with influence from OECD and NCTM). Approximatively, they represent learning objectives on a higher level. An advantage that is stressed is their independence of content and independence of students' age. The competencies are:

– `Think mathematically` includes the abilities to understand and handle mathematical concepts, their scope, and to understand and distinguish between different kinds of mathematical statements.
– `Argue mathematically` includes the abilities to develop and assess chains of arguments, to know what a mathematical proof is, to describe and reason about solutions, to uncover basic ideas in a line of arguments, and to understand reasoning and proof as fundamental aspects of mathematics.
– `Solve problems mathematically` includes the abilities to identify, pose and specify problems, to self-constitute problems, to monitor and reflect on the process of problem solving, to endue strategies and heuristics, and to solve different kinds of problems.
– `Model mathematically` includes the abilities to translate special areas and contents into mathematical terms, to work in the model, to interpret and verify results in the situational context, and to identify differences between the situation and the model.

– `Use mathematical representations` includes the abilities to understand and utilize different representations of mathematical objects, phenomena, and situations, to find relations between different representations, and to choose the appropriate representation for the special purpose.
– `Language` includes the abilities to use parameters, terms, equations and functions to model and interpret, to translate from symbolic and formal language into natural language and vice versa, and to decode and interpret mathematical language and understand its relations to natural language.
– `Communicate` includes the abilities to explain solutions, to use a special terminology, to work in groups, e.g. explain at an adequate level and understand and verify statements of others.
– `Use tools and aids` includes the abilities to know about the existence of various tools and aids for mathematical activities, their range and limitations, and to reflectively use them.

3 New Definition of the Competency Taxonomy

We define *elementary competencies* as pairs of a cognitive process and a knowledge element. We consider it a as a pair c of cognitive process p and a knowledge element k, $c = (p, k)$. Cognitive processes are defined as in [6] while knowledge elements represent facts, topics, concepts, theorems, rules/procedures and *Grundvorstellungen* [12] - i.e., elements of the knowledge dimension as available in ActiveMath and in the ontology of instructional objects OIO [10].

Composite competencies are defined as a set of multiple elementary competencies. In comparison to the work of van Assche [11], where a competency is defined as a tuple $c =< v, \{t_1, \ldots, t_n\} >$ (t_i are topics – our knowledge elements). Our definition, however, facilitates the representation within the learner model by reducing dimensionality without loss of expressivity.

3.1 Knowledge Ontologies / Extended Domain Ontologies

Knowledge elements can be related to each other, and therefore, a domain can be represented by an ontology. The knowledge includes concepts (e.g. fraction, integer, numerator), rules (e.g. addition of fractions with unlike denominators or subtraction), and *Grundvorstellungen*[6]. Grundvorstellungen in the fraction domain are part-whole, ratio, operator, quotient, and measure which provide different interpretations of a fraction in application contexts [13]. Other Grundvorstellungen exist for addition, multiplication and division of fractions too. The Grundvorstellungen have corresponding elements (nodes) in the educational domain ontology.

[6] We use the German term here since we could not find an appropriate translation for this term which was coined by German educationalists. A possible translations may be "interpretation/meaning of a concept".

3.2 Ontology of Cognitive Processes

According to Flavell [15], meta-cognition is composed of meta-cognitive knowledge and meta-cognitive experiences or regulation. Meta-cognitive knowledge includes acquired knowledge about cognitive processes. Flavell divides meta-cognitive knowledge into three categories: knowledge of person variables, task variables and strategy variables. Correspondingly, Anderson et al. [6] differentiate between *self-knowledge*, *knowledge about cognitive tasks*, and *strategic knowledge*, and place these meta-cognitive aspects exclusively into the knowledge dimension.

Our ontology of cognitive processes modifies and extends [6] by adding meta-cognitive processes that aim at representing the meta-cognitive regulation processes.

Although meta-cognitive competencies may be seen as more global/general proficiencies of a learner, the evidences always occur in a knowledge context whose influence has still to be investigated empirically. It may be that the ability to apply meta-cognitive processes varies depending on the proficiency in the respective domain and possibly even between different knowledge elements. Adhering to our definition of competency as pair of cognitive process and knowledge, the absence of meta-cognitive processes leads to meta-cognition related competencies, which can only express a global proficiency regarding meta-cognitive aspects and does not allow for a differentiation of how well a student is able to consciously apply meta-cognitive operations to specific knowledge elements or within a certain domain, e.g. questions such as "how proficient is the student in detecting errors within problems of the fraction domain?", cannot be answered. Therefore, we add meta-cognitive processes to the process dimension.

The first and second columns in Table 1 show the proposed hierarchy of the cognitive processes. Several cognitive processes are combined into categories:

- `Remember`: consists of the most basic retrieval operations performed on knowledge, i.e. the *recognition* of knowledge and its *recall* from memory.
- `Represent`: includes the abilities to *interpret* knowledge (e.g. "a fraction consists of two numbers: the numerator divided by the denominator"), to *illustrate* - to find an instance of a given concept, to *transform* from one representation to another, and to *summarize* (generalize) by inferring common principles or by identifying the main aspects of some information.
- `Solve`: includes the ability to *estimate* a result without calculating its exact value (e.g. estimate whether the addition of two fractions results in more or less than one), to *apply algorithms* with all their steps, and to *apply tools* appropriately (e.g. a calculator to add fractions).
- `Analyze`: summarizes abilities needed to break information into parts and to determine how these parts relate to each other and to the general picture. This category consists of the abilities to *check* information for inconsistencies or problems, to *differentiate* between important and unimportant information, to *organize* information according to some criteria, and to *attribute* a bias, value or intent to some presented material.

– `Model`: combines abilities needed to understand and create models in a specific domain. Included are the capability to *decode* information presented and transform it into a mathematical model (e.g. a textual description of ratios of some persons' ages, decoding could be putting the ratios into an equation), to *encode* a mathematical model into a situational context or its transformation into another domain. Furthermore, the category includes to *generate* hypotheses, and to *produce* new models by combining hypotheses to achieve a certain goal.
– `Communicate`: is concerned with explaining and discussing knowledge. It contains the processes of how to *describe* one's own knowledge, to *argue* about different aspects of some knowledge, and to *prove* certain facts.
– `Meta-cognition`: consists of meta-cognitive processes, i.e. processes that aim at reflecting and controlling cognitive processes. Such processes include to *reflect* upon one's own knowledge and thinking processes. Furthermore, *help seeking*, and to *search for information* to fill gaps in or extend one's own knowledge, to *detect errors* in one's own or the work of others, to *plan* tasks by dividing them into steps and order them according to their sequence of execution/implementation, to *self-monitor* one's own actions and behavior (e.g. by analyzing progress and differentiating between more effective learning strategies and less effective ones, and hence choose an appropriate one), and finally, to *self-explain*.

Some of the processes, such as *estimate*, could be placed into several of the categories, since they can be applied at different levels. The reason to place estimate into the `Solve` category is a specific interpretation, i.e., in the sense of applying rules of thumb or heuristics in order to get an idea of what the actual result may be. Alternatively, *estimate* may be interpreted as the process of ordering some aspects and inferring certain points.

4 Usage of the Ontologies

From the beginning, knowledge elements from domains have been characterized by metadata in ActiveMath. Exercises are annotated with metadata that specify which concepts they train and what cognitive processes are involved in attaining the correct solution. Cognitive processes have been present in ActiveMath as metadata for exercises, exercise steps and other learning objects. The metadata scheme evolved over time:

In the first version of the ActiveMath platform [16], Bloom's taxonomy was used. Later, we introduced the PISA-competencies for the LeActiveMath application of the ActiveMath-platform. The knowledge dimension is implicitly defined in the content and its metadata. This revision of competencies was driven by the influence of the pedagogical partners involved in the LeActiveMath project. Currently, ActiveMath relies on the competencies described above.

4.1 Student Modeling

ActiveMath's student model [17] can be parameterized to use different competency taxonomies. The structure of the student model includes nodes k_i for knowledge components. In case of using the proposed taxonomy, it stores pairs of cognitive processes and (domain) knowledge. This is done by relating each knowledge (super-)node k_i to a cognitive process p_j. The knowledge elements are dynamically extracted from the learning objects included in the content.

The student model derives competency values from evidences: exercises (whose metadata specify the knowledge and cognitive processes needed) and the respective performance/action of the student. Exercises-to-concept and concept-to-concept relations are dynamically extracted from the learning content in order to derive the competencies that have to be updated. In case of a competency involving multiple knowledge elements, evidences about proficiencies are equally attributed to all elementary competencies, as long as no further information is available to differentiate the attribution to a specific competency.

Evidences can be propagated along *prerequisite relations* in the student model. The propagated evidences are regarded as indirect evidences and are overridden as soon as direct evidence is available. In order to estimate proficiencies at a more general level, the hierarchical nature of the competency taxonomy is exploited, so that a more general competency reflects how the subcompetencies are mastered (and to what degree).

4.2 Selection of Content

Based on the estimations of the student model, the Tutorial Component selects appropriate learning objects, which serve the improvement of goal competencies [18]. One of the limitations of adaptively assembling courses is the availability of learning objects with perfectly matching knowledge and cognitive processes metadata. However, if no learning object is found that has exactly the metadata requested by the Tutorial Component, the hierarchical structure of both ontologies can be used to relax the search for learning objects and select a learning object that serves the training of a more general (or similar) cognitive process and knowledge.

Similarly, the two hierarchies (for knowledge and for cognitive processes) can be employed to facilitate mappings of exercises from different learning object repositories for course generation [19].

The relaxed search for knowledge and cognitive process metadata can lead to *approximate* mappings. Additionally, approximate mappings can be used for the alignment of different curricula as proposed in [11].

5 Relation to Existing Competency Taxonomies

One of the goals of the proposed competency taxonomy is to subsume and integrate existing competency systems, such as the PISA competency hierarchy and the revised Bloom taxonomy. Therefore, we define (partial) mappings between our cognitive process hierarchy, the revised Bloom taxonomy and the PISA competency hierarchy.

Table 1 relates our process hierarchy to the PISA (mathematics) competencies and to cognitive processes in [6]. In some cases, finding a corresponding PISA-competency is rather difficult. One reason for the difficulty is that the PISA-competencies often include multiple processes in a single competency and sometimes also elements that we would place into the knowledge dimension. Thus, for the mapping, we compared the aspects of PISA competencies regarding the actual cognitive processes involved. Elements that are placed into parentheses capture only part of the scope of a proposed cognitive process.

Paquette [9] (kindly brought to our attention by a reviewer) presents a top-level ontology for competencies, and a competency taxonomy pairing *generic skills* with *ressources*, corresponding to cognitive processes and knowledge elements respectively. His taxonomy combines and extends several previous approaches such as Bloom's. A mapping to the original approaches is provided.

6 Future Work

Since the proposed competency taxonomy leads to a fine granularity of competencies, ways to exploit the hierarchy of the taxonomy need to be developed in order to overcome the problem of sparse data. Therefore, we plan to explore the influence of the hierarchy on learner model estimations by comparing estimations with real students' performance. The additionally inferred data may provide a means for fine-grained and accurate estimations and, e.g. provide teachers with detailed information about students' weaknesses and strengths enabling them to revise their courses, e.g. to add an extra repetition of a poorly understood topic.

7 Conclusion

Most previous ITS have only used the knowledge dimension for characterizing their exercises and exercise steps as well as for building a structure for student models.

In order to make ITS-content and its metadata reusable and systems interoperable on learning objects, we propose a framework for competencies that can be used across domains and for many ITSs. This ontology framework includes two taxonomies, one for (domain) knowledge and one for cognitive processes. We define a taxonomy for cognitive processes which extends and modifies the taxonomy of Anderson et al. and we also extend the types of knowledge included in domain ontologies by Grundvorstellungen which are an important ingredient for real world problems.

We compare and translate several well-known competency taxonomies with the new ontological framework and briefly indicate why the hierarchical, two-dimensional framework is useful.

Table 1. Mappings of the proposed cognitive processes to revised Bloom and PISA.

Proposed category	Proposed process	PISA	Revised Bloom
Remember	Recognize	n/a	Recognize
	Recall	n/a	Recall
Represent	Interpret	Represent/Think	Interpreting
	Exemplify	Represent	Exemplifying
	Transform	Represent	Interpreting
	Summarize	Think	Summarizing
Compare	Find commonalities	Think	Comparing
	Find differences	Think	Comparing
	Classify	Think	Classifying
	Infer	Think	Inferring
	Order	Think	Comparing
Solve	Estimate	Solve	Inferring
	Apply algorithm	Solve	Executing
	Apply tool	Tools	n/a
Analyze	Check	Solve	Checking
	Differentiate	Think/Model	Differentiate
	Organize	n/a	Organize
	Attribute	(Think/Argue)	Attribute
Model	Decode	Model	Interpreting
	Encode	Model	Interpreting
	Generate	Model/Think	Generating
	Produce	n/a	Producing
Communicate	Describe	Communicate	Explaining
	Explain	Communicate	Explaining
	Critique	Argue	Critiquing
	Prove	Argue	Inferring
Meta-cognition	Reflect	(Solve/Model)	n/a
	Help seeking	n/a	n/a
	Search for information	n/a	n/a
	Detect errors	(Solve)	Checking
	Plan	(Solve)	Planning
	Self-monitor	(Solve)	Checking
	Self-explain	(Argue/Communicate)	(Explaining)

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