

# How ActiveMath Supports Moderate Constructivist Mathematics Teaching

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**Abstract:** ActiveMath is a Web-based intelligent e-Learning system for mathematics that can support a moderate constructivist teaching: it supports students with an amount of guidance and leaves sufficient space for the learner's own choices, reflection and activities; a student can choose one or several learning goals, a context and a learning scenario and for which ActiveMath generates personalized learning material; at places in the learning material, the learner can extend the book according to her needs; the student can engage in truly interactive problem solving (an input editor eases the input of mathematical expressions). Backend-engines support the diagnosis of the user's input. When the student makes a mistake while exercising, feedback is given and hints can be requested. In addition, the learner can consult the semantic lexicon, she can acquire printed material, interact with an interactive tools.

## 1.0 Introduction

New technology offers new chances but does not automatically lead to better learning opportunities. For instance, multimedia seems to be useful for learning and can be attractive through new forms of interactivity and communication. However, the multimedia presentation can also lead to less learning in case it is not designed and used appropriately.

In this article, we describe the e-Learning system ActiveMath and how its features support the pedagogical approach of moderate constructivist teaching (cf. Herzig, 1999) which has a solid empirical basis (Reinmann and Mandl, 2001).

To begin with, we describe the moderate constructivist perspective and recapitulate why it is considered to be valuable for the learning of mathematics. Then, we briefly describe some parts and components of ActiveMath that can be used to realize this teaching approach.

## 2.0 The moderate constructivist learning approach

From a constructivist perspective learning is considered to be an active, self-guided, constructive, situated and social process. During such a learning process information is selected, organised and in integrated into the available cognitive structure. Therefore, knowledge acquisition not only occurs by copying reality but succeeds by the learners' independent construction efforts.

Accordingly the learner plays an active role in the learning process. He is regarded to be an information-processing individual; the external stimulus is processed actively and independently. The kind and quality of the processing varies between the learners, depending on the account of different experiences, previous knowledge and levels of development of the learners. In contrast the teachers have to focus their preparatory activities mainly, while they abstain in the original teaching situation and perform only in case assistance is needed. The teacher is an initiator of and an adviser in the learning process.

Of course it is not possible to completely abandon instructions from lessons. Hence, the moderate constructivist theory has developed as a pragmatic approach which integrates

instructions into a theory that has a clear constructivist tendency. The moderate constructivist approach aims at using only few instructions and only does this where they are considered to be beneficial for the learning process and better than any alternative. They get their eligibility, as far as they serve as forms of suitable assistance or feedback.

In a number of ways we illustrate the realization of the pedagogical approach in ActiveMath.

### 3.0 The ActiveMath learning environment

ActiveMath is learning environment on the Web. It supports learning mathematics by an advanced content presentation based on a semantically encoded content. This is combined with an interactive exercise system that can evaluate the learner's input by using a mathematical system. ActiveMath models the competencies of the learner and updates this model by tracing its actions. This model is used by the tutorial component of ActiveMath which offers the service to create books according to a usage scenario (such as a *discovery* or an *exam simulation*) or to suggest changes in the books to better fit the learner.

More information about ActiveMath can be found in (Melis et al. 2006) and in the references quoted there. An online demonstration is available at <http://www.activemath.org/>

### 4.0 Supporting features in ActiveMath

In this section chosen exemplary features of ActiveMath that support the moderate constructivist approach are illustrated in a potential usage with a learner who wants to learn about derivation rules.

Our fictive learner is exploring about the product rule for derivation of real functions. Using the search function he can identify the theorem he wishes to study and can request a course to be produced by the system that fits a learning scenario.

The student solves very quickly first exercise in which he applies the product rule has to be applied. After that he is invited by the tutorial component to add into his book page a more challenging exercise such as the computation of the derivative of a product of polynomials of high degree (a randomly produced exercise is presented in figure 1).

Reading an example application of the product rule to  $x \mapsto x \cdot (x+3)^2$ , the student wonders about the derivative of a similar expression  $x \mapsto x \cdot (x+3)^2$ . He can copy and modify the formula from the “book” and request to compute its derivative to the integrated computer algebra system. To better view the function, he can copy functions to a plotter.

The screenshot shows a browser window with the URL <http://learn-calculus.activemath.org>. The page title is "ActiveMath - Exercise - Mozilla Firefox". The main header features the "Le Math Active" logo and the word "Exercise". Below the header, there is a warning icon, a pencil icon, and the title "Application of the product rule" followed by five yellow stars. The exercise text asks to "Compute the derivative of  $f(x) = (10 \cdot x^9 - 6 \cdot x^8 + 10 \cdot x^{30}) \cdot (3 \cdot x^7 - (-8) \cdot x^{50} + 4 \cdot x^5)$ ". Below this, a text input field contains the derivative formula:  $f'(x) = (90 \cdot x^8 - 48 \cdot x^7 + 30 \cdot x^{29}) \cdot (3 \cdot x^7 - (-8) \cdot x^{50} + 4 \cdot x^5) + (10 \cdot x^9 - 6 \cdot x^8 + 10 \cdot x^{30}) \cdot (21 \cdot x^6 - 40 \cdot x^{49} + 20 \cdot x^4)$ .

Figure 1: First step into a difficult derivation

http://learn-calculus.activemath.org - ActiveMath Applet:...

**Proof of the product rule**

$$f(x) = u(x) \cdot v(x) \quad \Rightarrow \quad f(x_0) = u(x_0) \cdot v(x_0)$$

$$\Rightarrow \frac{f(x) - f(x_0)}{x - x_0} = \frac{u(x) \cdot v(x) - u(x_0) \cdot v(x) + u(x_0) \cdot v(x) - u(x_0) \cdot v(x_0)}{x - x_0}$$

$$\Rightarrow \frac{f(x) - f(x_0)}{x - x_0} = \frac{(u(x) - u(x_0)) \cdot v(x) + u(x_0) \cdot (v(x) - v(x_0))}{x - x_0}$$

$$\Rightarrow \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0} \left( \frac{u(x) - u(x_0)}{x - x_0} \cdot v(x) + u(x_0) \cdot \frac{v(x) - v(x_0)}{x - x_0} \right)$$

$$\Rightarrow \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0} \left( \frac{u(x) - u(x_0)}{x - x_0} \cdot v(x) \right) + \lim_{x \rightarrow x_0} \left( u(x_0) \cdot \frac{v(x) - v(x_0)}{x - x_0} \right)$$

**Next Step**    **Description and explanation**  
 Due to the limit rule for sums, we may form the limit separately for each of the two summands (the limit of a sum is the sum of the limits of each summand).

**Replay**

**Replay all**

**Figure 2: Animated proof of the product rule**

While working, the learner model gathers evidence about the competencies of our learner for each concept linked to her activities. The learner can thus track the progress of the learning process and the progress is indicated by a color code in the table-of-contents.

Trying to understand the formal proof, he can view an animated proof which provides a tangible presentation. This is depicted in figure 2.

At a later stage, when revising these rules, he wishes to obtain an overview of the derivation rules; from the *overview* content element at the end of the chapter about this topic, he can navigate to statements and examples of the individual rules. During his review, he will try to reconnect mentally one to each other, e.g. the fact that some proofs of the product rule rely on the quotient rule (and conversely). These connections can also be expressed in a graphical way using the concept-mapping tool and have these connections, entered by hand, be checked against the domain model.

Moreover the student his own “book” by dragging items from existing “books” or other websites.

## 4.0 Conclusion

In this paper we have described the approach of the ActiveMath learning environment to use ICT to support learning processes following moderate constructivism. The system is unique in the sense that it presents informative learning resources and interactive exercises with immediate feedback, as well as tools (CAS, interactive concept map, assembly tool, semantic search, function plotter, open-learner-model) into one coherent interface.

The system supports the learner by tracing her activities, diagnosing and modelling her competencies, and providing suggestions to organize her learning experience.

ActiveMath is undergoing large-scale evaluations in schools in Germany and Universities in Spain and Scotland. Preliminary evaluations have seen students qualifying ActiveMath as *tool to learn math which is superior to tools they have been using*.

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